Lab 1

performed 2025-01-27

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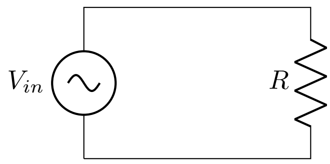
1 Introduction

Unlike DC signals, AC signals are time-varying, posing unique challenges to recording, characterizing and otherwise studying them.

This lab examines the circuit depicted in fig. 1, and focuses on comparing the measurement of one aspect of an AC signal – the RMS voltage – as seen by two tools: the Digital Multimeter (DMM), and the oscillscope. Numerical and analytical methods are used to model this circuit and derive expected RMS values for comparison.

Figure 1 depicts the simple circuit under study:

Figure 1: The simple circuit



Multiple variations on three different types of AC signals are provided at V_{in}.

A resistor was selected with value $R = 7.5\,\mathrm{k}\Omega$, with a 5% precision band; note that the exact resistance value of the resistor really isn't important, since its characteristics are not under study; it is only important that its impedance be much larger than the 50-ohm internal resistance of the function generator in use.

2 Analytic Modeling Results

Three different types of AC signals are provided at V_{in}:

1. A sinusoidal signal with no DC offset, at two different frequencies and magnitudes
2. The same sinusoidal signals as above, but with a small positive DC offset
3. A square wave voltage, at 25% and 50% duty cycles, with two different AC magnitudes.

For a signal $x(t)$, we are given the following formula for its RMS voltage:

$$X_{RMS} = \sqrt{\frac{1}{T}\int x(t)^2 dt} \tag{1}$$

2.1 Signal type 1 (sinusoidal AC)

Given frequency ω (rad./s) and magnitude (peak amplitude) V_m, signal type 1 can be modeled as:

$$v_1(t) = V_m \cos(\omega t) \tag{2}$$

(One can arrive here from the general form, $V_m \cos(\omega t + \theta)$, by shifting the beginning and end of the measurement window by $-\theta$, as the oscilloscope will do when it measures RMS voltages from a peak-to-peak cycle.)

Substituting v_1 into equation 1, the RMS voltage V_{1RMS} can be expressed as:

$$\begin{aligned} V_{1RMS} &= \sqrt{\frac{1}{T}\int_0^T V_m^2 \cos^2(\omega t) dt} \\ &= V_m \sqrt{\frac{1}{T}\int_0^T \frac{1}{2} + \frac{\cos(2\omega t)}{2} dt} \\ &= V_m \sqrt{\frac{1}{T}\left[\frac{t}{2} + \frac{1}{4\omega}\sin(2\omega t)\right]_0^T} \\ &= V_m \sqrt{\frac{1}{T}\left(\frac{T}{2} + \frac{1}{4\omega}\cancel{\sin(2\omega T)} - \frac{1}{4\omega}\cancel{\sin(0)}\right)} \\ &= V_m \sqrt{\frac{1}{\cancel{T}}\frac{\cancel{T}}{2}} \\ &= \frac{V_m}{\sqrt{2}}. \end{aligned} \tag{3}$$

2.2 Signal type 2 (sinusoidal AC with DC offset)

Given frequency ω (rad./s), peak amplitude V_m and DC offset V_b, we can model signal type 2 as:

$$v_2(t) = V_m \cos(\omega t) + V_b \tag{4}$$

Thus, substituting v_2 into equation 1, the RMS voltage V_{2RMS} can be expressed as:

$$
\begin{aligned}
V_{2RMS} &= \sqrt{\frac{1}{T}\int_0^T \left(V_m\cos\left(\omega t\right) + V_b\right)^2 dt} \\
&= \sqrt{\frac{1}{T}\left(V_m\right)^2\int_0^T \cos^2\left(\omega t\right) + 2\frac{V_b}{V_m}\cos\left(\omega t\right) + \left(\frac{V_b}{V_m}\right)^2 dt} \\
&= V_m\sqrt{\frac{1}{T}\int_0^T \frac{1}{2} + \frac{\cos\left(2\omega t\right)}{2} + 2\frac{V_b}{V_m}\cos\left(\omega t\right) + \left(\frac{V_b}{V_m}\right)^2 dt} \\
&= V_m\sqrt{\frac{1}{T}\left[\frac{t}{2} + \frac{1}{4\omega}\sin\left(2\omega t\right) + \frac{2V_b}{\omega V_m}\sin\left(\omega t\right) + t\left(\frac{V_b}{V_m}\right)^2\right]_0^T} \\
&= V_m\sqrt{\frac{1}{T}\left(\frac{T}{2} + \cancel{\frac{1}{4\omega}\sin\left(2\omega T\right)} + \cancel{\frac{2V_b}{\omega V_m}\sin\left(\omega T\right)} + T\left(\frac{V_b}{V_m}\right)^2 - \cancel{\frac{1}{4\omega}\sin\left(0\right)} - \cancel{\frac{2V_b}{\omega V_m}\sin\left(0\right)}\right)} \\
&= V_m\sqrt{\frac{1}{\cancel{T}}\left(\cancel{T}\right)\left(\frac{1}{2} + \left(\frac{V_b}{V_m}\right)^2\right)} \\
&= V_m\sqrt{\frac{1}{2} + \left(\frac{V_b}{V_m}\right)^2} \\
&= \sqrt{\frac{V_m^2}{2} + V_b^2}.
\end{aligned} \tag{5}
$$

Note: I won't show it here, but general, for a periodic signal made of two components $v = v_a + v_b$, where the two components are of different frequencies, it can be shown from Equation 1 that the RMS voltage V_{RMS} of the combined signal is equal to the root of the sum of the squares of the RMS voltages of its additive components, i.e.

$$V_{RMS} = \sqrt{{V_{aRMS}}^2 + {V_{bRMS}}^2}.$$

2.3 Signal type 3 (square wave)

Given magnitude V_m, frequency $\frac{1}{T}$ (Hz), and duty cycle D, our square wave (type 3) can be modeled as:

$$0 < t \leq T, \quad v_3(t) = \begin{cases} V_m & 0 \leq t < DT \\ -V_m & DT \leq t < T \end{cases} \tag{6}$$

This time, the integration is trivial: at any time, $v_3(t)$ is either V_m or $-V_m$, so $(v_3(t))^2 = {V_m}^2$:

$$V_{3RMS} = \sqrt{\frac{1}{T}\int_0^T (V_m)^2 dt} = \sqrt{\frac{1}{\cancel{T}}\cancel{T}{V_m}^2} = V_m. \tag{7}$$

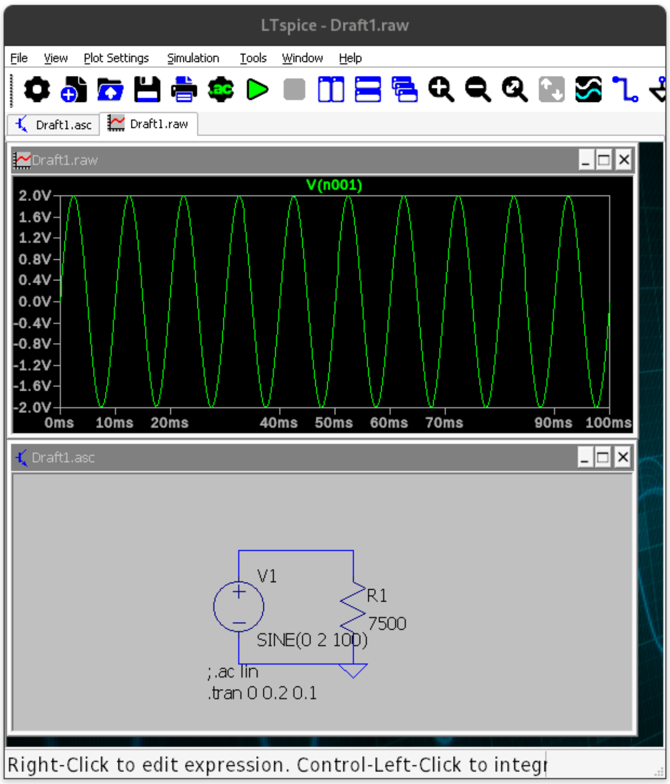
These three derivations are re-referenced in the "Experimental Results" section below.

3 Numerical Modeling Results

For the numerical modeling, I opted to simulate signals types 1 (sinusoidal AC about 0V) and 3 (square wave).

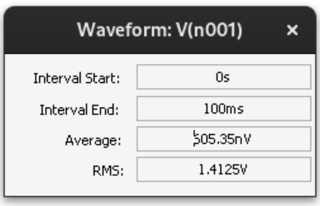
In LTSpice, I assembled the circuit shown in Figure 2; for the first signal, I specified a transient simulation from 0.1s to 0.2s:

Figure 2: Our first sinusoidal signal circuit, simulated in LTSpice



I then used Ctrl+click on the signal label `V(n001)` to pull up the Waveform dialog shown in Figure 3, yielding a numerically-derived RMS voltage:

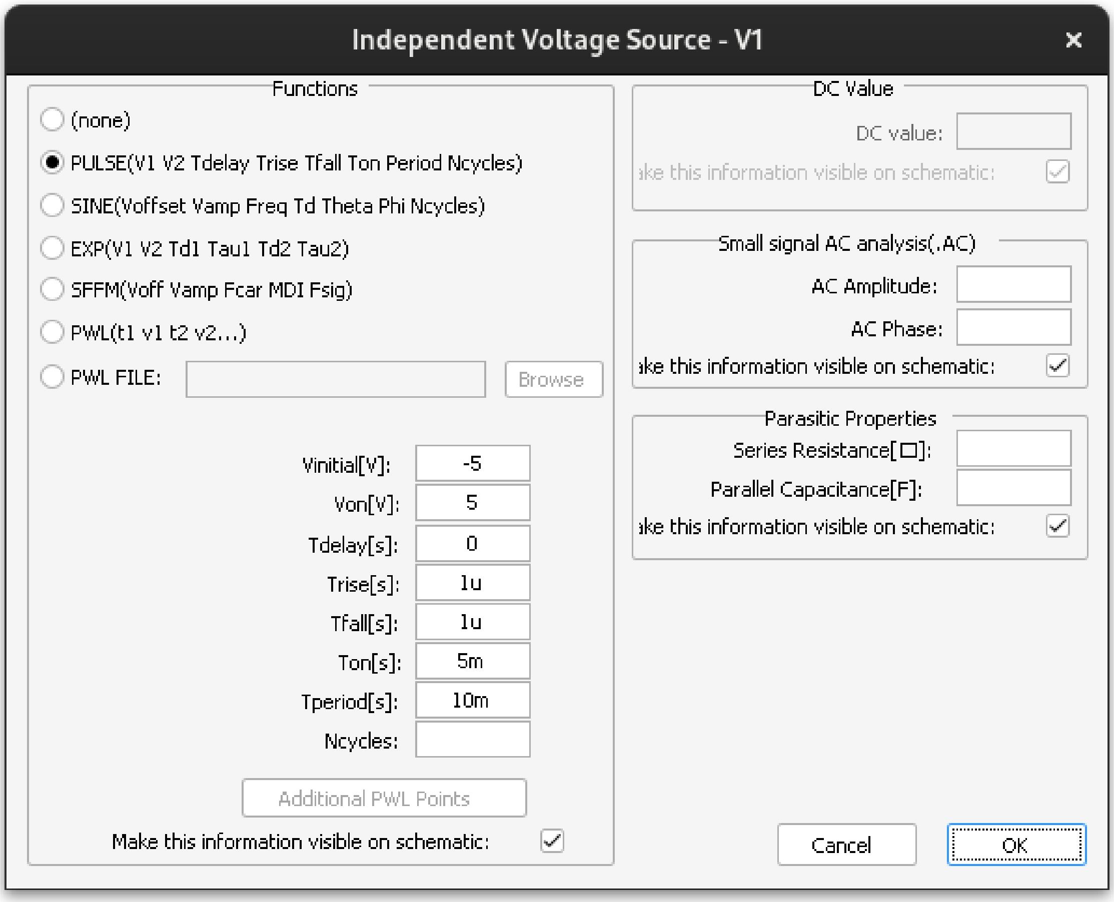
Figure 3: This LTSpice dialog shows us measurements of our interval; of most interest is RMS



I had to adjust the size length of the transient simulation to get an easily-viewable result.

For the square-wave values: I switched from the `SINE` command to `PULSE`; this command requires a rise-time and fall-time, which are set as low as possible to mimic a true square-wave; in addition, we specify the duty cycle and frequency indirectly, instead by specifying the on time and period, as seen in Figure 4.

Figure 4: The LTSpice `PULSE` command menu



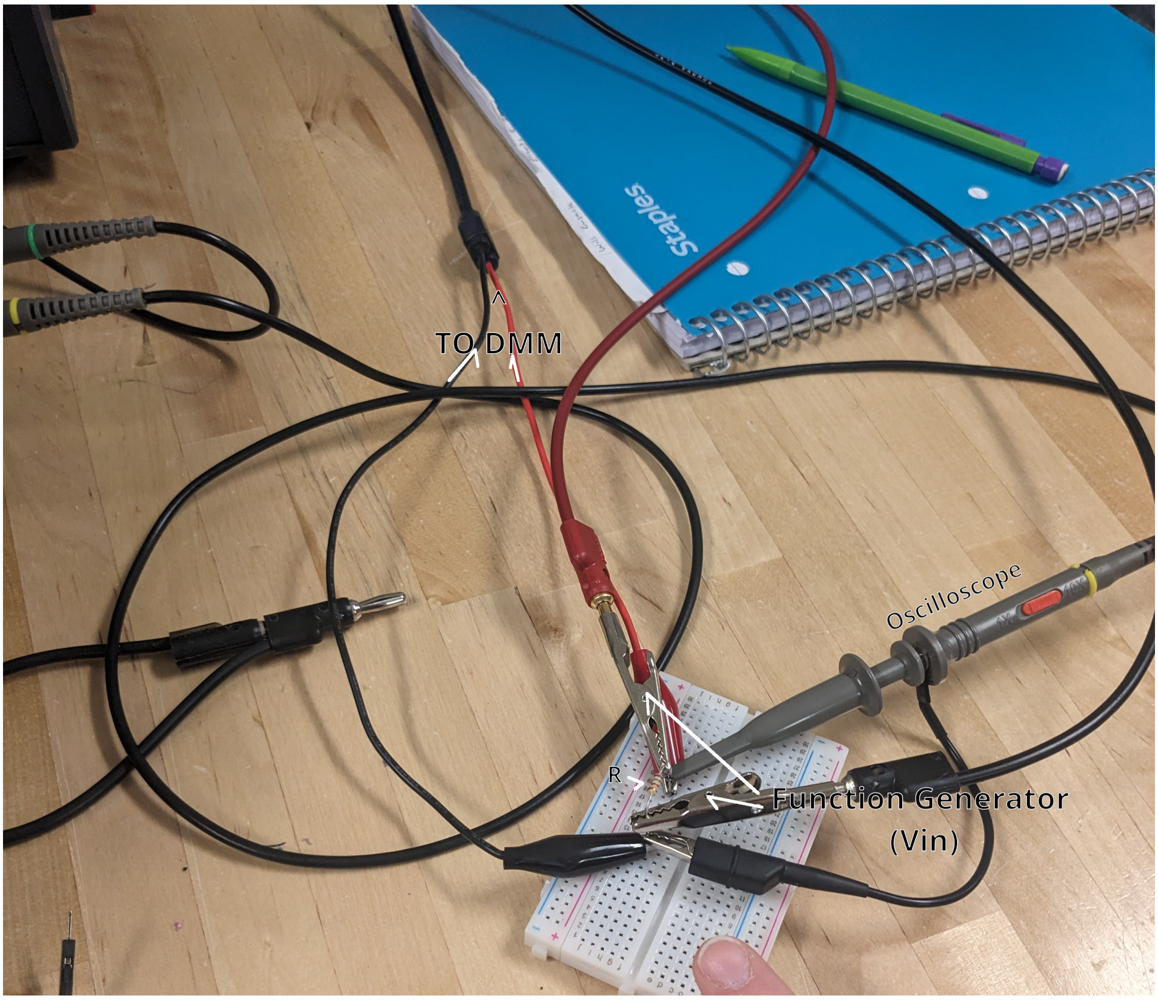
The results of the numerical modeling are compiled below, in Tables 7 and 8.

4 Experimental Results

The circuit we implemented can be seen in Figure 5. This configuration connects one resistor leg with the signal lead of the oscilloscope, the positive lead each of the function generator and DMM; repeat the same with the other resistor leg, the ground lead of the scope, and the negative leads of each the DMM and function generator. In effect, all pieces of equipment are placed in parallel, consistent with any other procedure for measuring the facets of a signal's voltage.

(Our circuit builder was Peyton; our checker was Will; I was grouped with these two as there

Figure 5: Breadboard, with resistor R connected to our DMM, scope and function generator



were an odd number of students.)

This configuration was used for the entire lab procedure, and adjusted both our function generator and oscilloscope through the variations of the three different signal types; in all three cases, we first used the oscilloscope to read the period and magnitude of the signal, and then used the DMM to measure the signal's RMS voltage.

Note: our use of the oscilloscope for magnitude measurements will later be identified as a key source of error.

4.1 Experiment 1 (sinusoidal AC)

Given a read period of T seconds, we calculate the frequency as $\frac{1}{T}$ Hz. For the RMS voltage, we use the formula derived at Equation 3:

$$V_{1RMS} = \frac{V_m}{\sqrt{2}}$$

Table 1: Voltage measurements and period for 1 (sinusoidal AC)

Set Mag.	Set Freq.	Read Mag.	Read Period	Calc. Freq.	Calc. RMS	Meas. RMS
2V	100 Hz	2.10 V	9.994 ms	100.1 Hz	1.48 V	1.4236 V
2V	50 kHz	2.05 V	19.95 us	50.13 kHz	1.45 V	1.4112 V
5V	100 Hz	5.11 V	10.01 ms	99.90 Hz	3.61 V	3.5522 V
5V	50 kHz	5.11 V	20.01 us	49.98 kHz	3.61 V	3.5451 V

Comparing the calculated and measured RMS values:

Table 2: RMS Error for 1 (sinusoidal AC)

	Calc. RMS	Meas. RMS	Error %
Case 1	1.48 V	1.4236 V	3.81 %
Case 2	1.45 V	1.4112 V	2.68 %
Case 3	3.61 V	3.5522 V	1.60 %
Case 4	3.61 V	3.5451 V	1.80 %

4.2 Experiment 2 (sinusoidal AC with DC offset)

Given a read period of T seconds, we calculate the frequency as $\frac{1}{T}$ Hz. For this signal's RMS voltage, we use the formula derived at Equation 5:

$$V_{2RMS} = \sqrt{\frac{V_m^2}{2} + V_b^2}$$

Table 3: Voltage measurements and period for 2 (sinusoidal AC with DC offset)

Set Mag.	Set Freq.	DC bias	Read Mag.	Read Period	Calc. Freq.	Calc. RMS	Meas. RMS
2V	100 Hz	2V	2.13 V	10.00 ms	100.0 Hz	2.50 V	2.44 V
2V	100 Hz	-5V	2.11 V	9.996 ms	100.0 Hz	5.22 V	5.19 V
5V	100 Hz	2V	5.15 V	9.998 ms	100.0 Hz	4.15 V	4.05 V
5V	100 Hz	-5V	5.20 V	9.997 ms	100.0 Hz	6.21 V	6.16 V

Comparing the calculated and measured RMS values:

Table 4: RMS Error for 2 (sinusoidal AC with DC offset)

	Calc. RMS	Meas. RMS	Error %
Case 1	2.50 V	2.44 V	2.40 %
Case 2	5.22 V	5.19 V	0.57 %
Case 3	4.15 V	4.05 V	2.41 %
Case 4	6.21 V	6.16 V	0.81 %

Table 4: RMS Error for 2 (sinusoidal AC with DC offset)

4.3 Experiment 3 (square wave)

Given a read period of T seconds, we calculate the frequency as $\frac{1}{T}$ Hz. As we saw when deriving Equation 7, this signal's RMS voltage is the same as its magnitude.

Table 5: Voltage measurements and period for 3 (square wave)

Set Mag.	Set Freq.	Duty	Read Mag.	Read Period	Calc. Freq.	Calc. RMS	Meas. RMS
2V	100 Hz	25%	2.11 V	10.00ms	100.0 Hz	2.11 V	2.02 V
2V	100 Hz	50%	2.13 V	10.00ms	100.0 Hz	2.13 V	2.01 V
5V	100 Hz	25%	5.20 V	9.998ms	100.0 Hz	5.20 V	5.04 V
5V	100 Hz	50%	5.20 V	9.999ms	100.0 Hz	5.20 V	5.01 V

Comparing the calculated and measured RMS values:

Table 6: RMS Error for 3 (square wave)

	Calc. RMS	Meas. RMS	Error %
Case 1	2.11 V	2.02 V	4.27 %
Case 2	2.13 V	2.01 V	5.63 %
Case 3	5.20 V	5.04 V	3.08 %
Case 4	5.20 V	5.01 V	3.65 %

Notably, this is the first time we have a >5% error value; we will review this item in TODO WHERE?

5 Data Comparison

Here, I publish the modeling results for 1 (sinusoidal AC about 0V).

Table 7: RMS Voltage comparison for 1 (sinusoidal AC)

RMS	Case 1	Case 2	Case 3	Case 4
Analytic (A)	1.48 V	1.45 V	3.61 V	3.61 V
Numerical (N)	1.4125 V	1.4124 V	3.5311 V	3.5311 V
Experimental (E)	1.4236 V	1.4112 V	3.5522 V	3.5451 V
A-N error	4.78 %	2.66 %	2.23 %	2.23 %
A-E error	3.96 %	2.75 %	1.63 %	1.83 %
N-E error	0.78 %	0.09 %	0.59 %	0.39 %

Next, I publish the modeling results for 3 (square-wave AC).

Table 8: RMS Voltage comparison for 3 (square-wave AC)

RMS	Case 1	Case 2	Case 3	Case 4
Analytic (A)	2.11 V	2.13 V	5.20 V	5.01 V
Numerical (N)	1.9999 V	1.9999 V	4.9997 V	4.9997 V
Experimental (E)	2.02 V	2.01 V	5.04 V	5.01 V
A-N error	5.51 %	6.50 %	4.01 %	4.01 %
A-E error	4.46 %	5.97 %	3.17 %	1.96 %
N-E error	1.00 %	0.50 %	0.80 %	1.97 %

6 Conclusions

Before completing the lab report out and making firm conclusions, I'd like to address our two analysis questions:

> PSpice: In the transient simulation profile: what is the role of "Maximum Step Size"? Create an example and include waveform images to illustrate your point.

Returning to our simulation of a 5V sinusoidal waveform with no DC bias at 50kHz: the larger we allow the "Step Size" to go, the fewer timesteps LTSpice will take when performing its numerical simulation, and thus the less precise our RMS value is / the less close it is to $\frac{5}{\sqrt{2}} \approx 3.53553....$ In Figures 6, 7, 8, and 9, we can see that restricting the timestep size down to 1 ns brings our RMS voltage much closer to $\frac{5}{\sqrt{2}} \approx 3.53553$.

As for the second question:

> Considering your experimental data, explain why we can conclude that the DMM is showing the true RMS regardless of the waveform.

A value for RMS voltage was arrived at through multiple different means; from this, it was discovered that no waveform yields a particularly higher error for RMS voltage than any other; in fact, more specifically, the highest error values are associated only with the analytical derivation, which relies on an oscilloscope-derived magnitude reading.

I'll review the oscilloscope-derived magnitude problem in a moment, but I wanted to finish my point regarding the suitability of the DMM to produce RMS voltage measurements: the lack of particularly high error values when comparing experimental (DMM) RMS values for any of the various waveforms indicates that the DMM's suitability does not, within reason, depend on the waveform it's measuring.

As for the error values: the largest cases of error are when comparing the experimental and numerical results to the analytical results for 2V square-wave AC. Despite setting the function generator to 2V, the oscilloscope nevertheless reads a peak-to-peak magnitude of 4.22 and 4.26V (so, a peak-magnitude 2.11V of 2.13V, respectively). This likely has to do with how the oscilloscope regisers peak-to-peak voltages, relying on hazy extremes; but also, notably, because we had our oscilloscope in 10X mode, reducing its signal sensitivity / resolution.

Figure 6: The LTSpice simulation with no defined “Maximum Step Size”

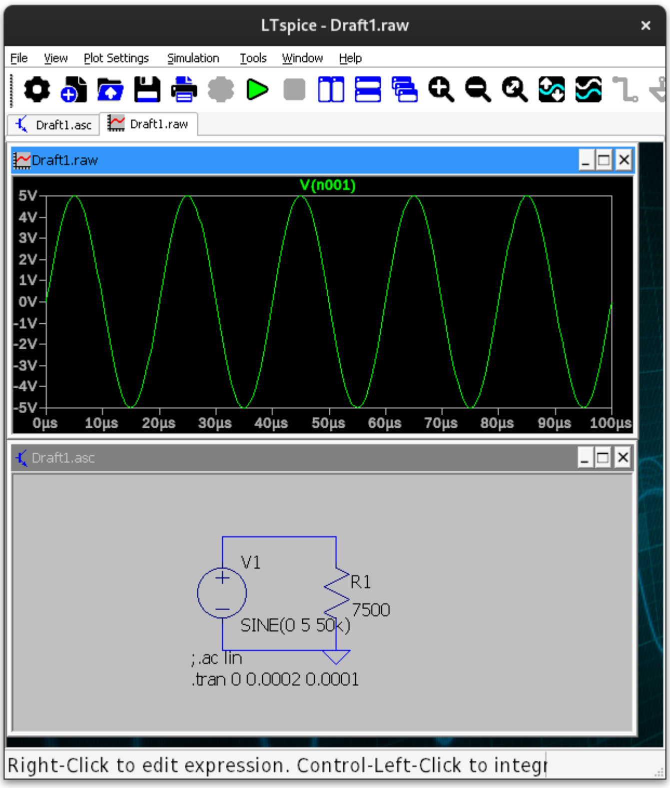


Figure 7: The RMS value of the simulation with no defined “Maximum Step Size”

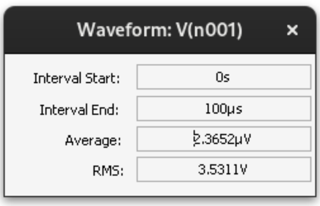


Figure 8: The LTSpice simulation with a “Maximum Step Size” of 1 ns

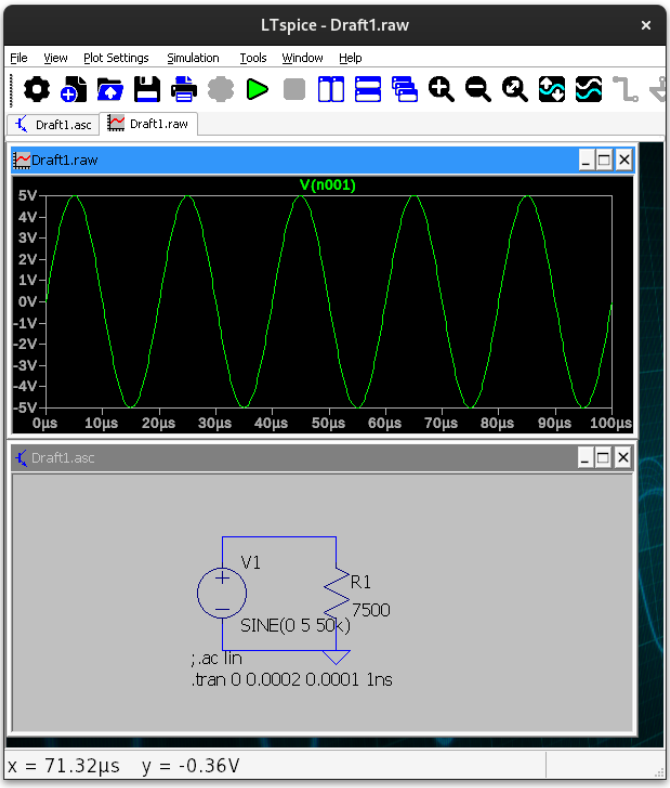


Figure 9: The RMS value of the simulation under a “Maximum Step Size” of 1 ns